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ESTIMATES OF ERRORS IN MISTRAM DUE TO ATMOSPHERIC REFRACTION

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ABSTRACT

The atmosphere causes errors in the measurement of range, range difference, range rate, and range difference rate. The magnitudes of these errors are estimated and found to exceed the maximum allowable errors for Mistram. Refractive index corrections must be applied to the Mistram System to reduce the errors to the same magnitude as the maximum allowable errors.

1.0 Introduction

The Mistram missile trajectory measurement system derives its output from simultaneous measurements of range by a central station and remote stations. The output consists of the missile range measured at the central station, R_o, and range differences, D_{jo}, determined from the relative times of arrival at the central station and the remote stations.

The atmosphere introduces errors in both R_o and D_{jo} by introducing changes in the velocity of propagation. The refraction errors included in the measurement R_o range from less than a meter to a few hundred meters depending upon the position of the missile relative to the central station. The magnitudes of the range errors do not appreciably increase with distance after the first twenty to thirty kilometers of atmosphere (altitude above mean sea level) have been traversed. This means the relative accuracy of the range measurement increases with range. The D_{jo} is essentially fixed by the elevation and azimuth angles of the missile relative to the site and is approximately independent of range. The resulting position error caused by errors in both R_o and D_{jo}, however, increases with increasing distance.

The specified accuracies of the Mistram system are listed in Table 1. These specified maximum errors include the average effects of the propagation medium. These maximum error specifications hold for elevation angles, α_o, between 5° and 85° and ranges from 55 km to 1100 km. If the specified errors are to be maintained, the errors due to propagation through the atmosphere must be corrected. The expected

daily variations of the average propagation errors also exceed the specified maximum errors. The refraction correction system used with Mistram must be able to correct for the changes of the atmosphere with time.

TABLE 1

SPECIFIED MAXIMUM ERRORS FOR THE MISTRAM SYSTEM*

Range error	ΔR_o	$< 1.2 \times 10^{-4}$ km
Range difference error	ΔD_{jo}	$< 9.2 \times 10^{-6}$ km
Range rate error	$\Delta(\frac{dR_o}{dt})$	$< 6.0 \times 10^{-6}$ km/sec.
Range difference rate error	$\Delta(\frac{dD_{jo}}{dt})$	$< 6.0 \times 10^{-7}$ km/sec.

2.0 Range Measurement Errors Due to Refraction

The Mistram system measures range, range difference, range rate, and range difference rate. To make these measurements, the Mistram system uses one central station and from two to four remote sites. The range is determined at the central station by measuring the time required for a signal to propagate from the central station to the missile and back. The range differences are determined by measuring the differences in the times required for the signal to propagate from the missile to the central station and remote sites.

The Mistram site configuration is depicted in Figure 1. The range error, R_o, is a function of the position of the target relative to the central

* Page 9, reference 1.

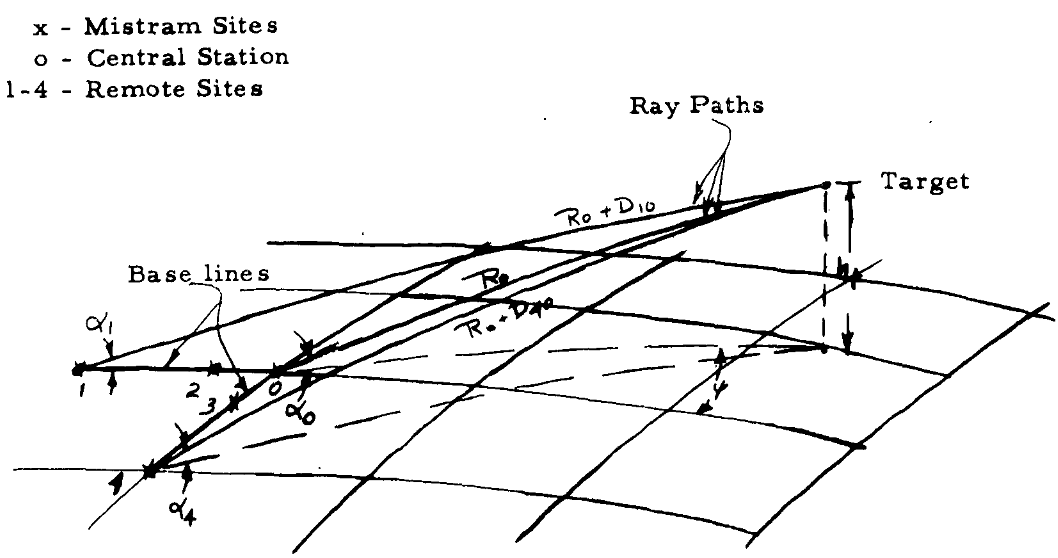

Figure 1

station and the index of refraction of the intervening atmosphere. The range error may be approximately calculated by (reference 2):

$$\Delta R_o \cong \frac{10^{-6}}{\sin \alpha_o} \int_{h_r}^{h_t} N(h)\,dh \quad \text{in km.} \tag{1}$$

where: $N(h) = (n(h)-1) \times 10^6$, n = index of refraction, and α_o = Elevation Angle

The approximation holds for $\alpha_o > 10°$ and a horizontally stratified index of refraction. On the average at Cape Canaveral the integral

$$I_o(\infty) = 10^{-6} \int_0^{\infty} N(h)\,dh = 2.5 \times 10^{-3}\,\text{km}.$$

and represents the range error for a vertical ray path through the atmosphere. The average expected range error is

$$\Delta R_o \cong \frac{I_o(\infty)}{\sin\alpha_o} = \frac{2.5 \times 10^{-3}}{\sin \alpha_o} \text{ km} = \frac{2.5}{\sin\alpha_o} \text{ meters} \qquad (2)$$

where the height of the target is greater than 30 km. For target altitudes less than 30 km, the range error will be less.

The form of the radio refractivity profile, N(h), is roughly an exponential decrease with height. For a given elevation angle, the error per unit path length decreases in an exponential fashion with height. The relative accuracies of the range errors then increase with range or height for a given elevation angle. For elevation angles greater than 10°, the angle α_o is approximately the elevation angle for a straight line between the target and the central station. For increasing ranges and constant elevation angles, the increase in total range error is negligible for altitudes greater than 30 km.

For elevation angles less than 10°, the range error cannot be accurately computed using equation (1). The exact equation for range error, assuming horizontal stratification, is (reference 3)

$$\Delta R_o = 10^{-6} \int_{h_r}^{h_t} \frac{N(h)\,dh}{\sin(\alpha_o + \phi - \tau)} + \int_{h_r}^{h_t} \frac{dh}{\sin(\alpha_o + \phi - \tau)} - R_{SR} \qquad (3)$$

where τ is the angular bending of the ray path, ϕ is the central angle (relative to the center of the earth) subtended by the central station and the target, and R_{SR} is the straight line distance from the central station to the target.

The ranges of values of ΔR_0 that may be expected may be estimated using the NBS exponential atmosphere (reference 4).

TABLE 2

RANGE ERRORS CALCULATED USING THE CRPL EXPONENTIAL ATMOSPHERE FOR TARGET HEIGHT OF 425 KM*

Elev. Angle	ΔR_0 for N_S=200	ΔR_0 for N_S=450
0°	65 m.	192 m.
3° 43'	21 m.	29 m.
11° 27'	9 m.	10 m.

A sampling of 84 radio refractivity profiles measured at Cape Canaveral were studied in detail by Thayer and Bean at NBS. The ranges of values calculated from these profiles are presented in Table 3.

The range errors given in Tables 2 and 3 are for an atmosphere whose index of refraction is horizontally stratified. Any real atmosphere is not horizontally stratified but includes tilted layers and local inhomogeneities such as clouds. The effects of the local inhomogeneities in the atmosphere may be investigated by using the stratified atmosphere calculations and adding correction terms. The effects of tilted layers may be investigated by moving the origin of the coordinate system. The range error may be expressed by (Reference 3)

$$\Delta R_0 \cong \int_{h_r}^{h_t} \frac{N(h)\,dh}{\sin \alpha} \cong \int_{h_r}^{h_t} \frac{N_0(h)\,dh}{\sin \alpha} + \sum_{k=1}^{M} \int_{r_{k_1}}^{r_{k_2}} \Delta N_k \, dR$$

* Extracted from tables in reference 4.

$$\Delta R_o \cong \int_{h_r}^{h_t} \frac{N_\sigma(h)\,dh}{\sin\alpha} + \sum_k \Delta N_k \left(r_{k_2} - r_{k_1}\right) \tag{4}$$

where the ΔN_k is the k^{th} deviation from the stratified atmosphere, $N_\sigma(h)$ refers to the assumed stratified atmosphere, and $r_{k_2} - r_{k_1}$ is the path length through the k^{th} local disturbance.

The magnitude of each disturbance is assumed small so it does not affect the total amount of bending along the path. For a cloud, the ΔN value is about 10 (reference 5). If a length of about 1 km is assumed, an increase in range error of about 0.01 meters may be expected. Since this is negligible compared to the total error accumulated, the effects of local inhomogeneities will only be noted (reference 6).

TABLE 3

RANGE ERROR CALCULATED FROM MEASURED PROFILES*

Elev. Angle	Ht.	$\Delta R_{min.}$	$\overline{\Delta R}$	$\Delta R_{max.}$
0°	1 km.	23 m.	41 m.	59 m.
	5 km.	46 m.	78 m.	107 m.
	70 km.	65 m.	107 m	145 m.
$2^\circ 51'$	1 km.	3.7 m.	5.8 m.	7.1 m.
	5 km.	14.5 m.	21.2 m.	24.9 m.
	70 km.	27.5 m.	38.7 m.	43.6 m.
$11^\circ 27'$	1 km.	0.96 m.	1.49 m.	1.84 m.
	5 km.	4.05 m.	5.81 m.	6.75 m.
	70 km.	8.6 m.	11.9 m.	13.1 m.

*Extracted from Table 1, pg. 4 of Reference 2.

The phenomenon of ducting will also increase range and range difference errors. The approximate amount of increase depends upon the target, site geometry and the atmospheric conditions. In general, ducting will not be a problem in the area specified for the operation of the Mistram system (references 6 and 7).

The refractive range errors vary between a part of a meter and two hundred meters depending upon the atmospheric conditions and the target, site geometry. The variation in range error may be as large as fifty percent of the average range error (Table 3). The maximum range error specified for the Mistram system is 0.12 meters. The expected errors and changes in errors due to atmospheric refraction are many times the maximum allowable error.

3.0 Range Difference Measurement Errors Due to Refraction

The refraction error included in the measurement of D_{jo} may be computed if the range errors for both the central station and the remote station are known.

$$\Delta D_{jo} = (R_j + \Delta R_j - R_o - \Delta R_o) - (R_j - R_o) = \Delta R_j - \Delta R_o$$

For elevation angles greater than 10^o,

$$\Delta D_{jo} = I_o(\infty)\left[\frac{1}{\sin \alpha_j} - \frac{1}{\sin \alpha_o}\right] \tag{5}$$

where the index of refraction profile is assumed to be horizontally stratified and identical for both ray paths.

For purposes of analysis, the target, site geometry may be reduced to the plane figure depicted in Figure 2. An effective baseline, B_{joe},

is introduced to compensate for the azimuth angle, ψ, shown in Figure 1. The maximum error, $\Delta D_{jo_{max}}$, occurs when the target is along the extended baseline or when $B_{joe} = B_{jo}$.

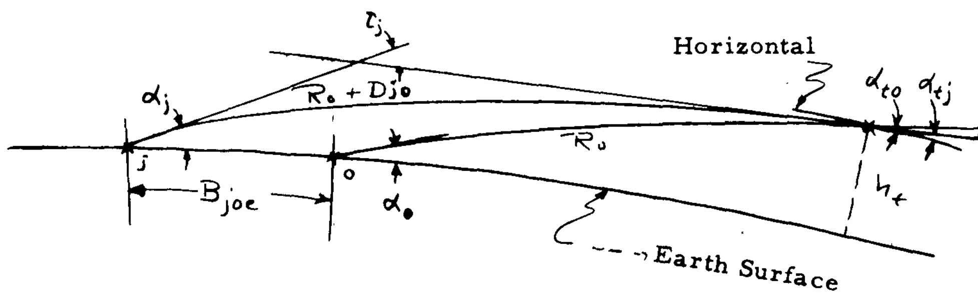

Figure 2

For large R_o the ray paths are approximately parallel. At the target the elevation of the ray relative to the horizontal is

$$\alpha_{to} = \alpha_o + \phi - \tau_o$$

$$\alpha_{tj} = \alpha_j + \phi + \phi_{je} - \tau_j$$

where $\phi_{je} = \dfrac{B_{joe}}{r_o}$, r_o = radius of the earth at MSL.

$$\therefore \alpha_o - \alpha_j = \phi_{je} + (\tau_o - \tau_j) + (\alpha_{to} - \alpha_{tj}) = \gamma_{je}$$

Using Equation (5)

$$\Delta D_{jo} \cong \frac{I_o(\infty)\,\gamma_{je}\cot\alpha_o \csc\alpha_o}{1 - \gamma_{je}\cot\alpha_o} \qquad (7)$$

For target altitudes greater than 30 km,

$$\tau_o - \tau_j < \phi_{je}$$

$$\alpha_{to} - \alpha_{tj} < \phi_{je}$$

As a crude approximation,

$$\alpha_o - \alpha_j = \gamma_{je} \cong \phi_{je}$$

$$\Delta D_{jo} \cong \frac{I_o(\infty)\ \phi_{je} \csc\alpha_o \cot\alpha_o}{1 - \phi_{je} \cot\alpha_o} \quad , \ \alpha_o > 10^\circ \tag{8}$$

For elevation angles less than 10°, the range error may be related to $I_o(h)$ by (reference 3)

$$\Delta R_j = \frac{1}{\sin \alpha e_j} \ I_o(h)$$

where the effective angle α_{ej} is a function of the target, site geometry and the radio refractivity profile N(h).

This equation may be used to define the angle α_{ej} given ΔR_j and $I_o(h)$.

$$\sin \alpha_{ej} \overset{\Delta}{=} \frac{I_o(h)}{\Delta R_j}$$

The effective angle for one path may be related to the effective angle for the other path by

$$\alpha_{eo} - \alpha_{ej} \cong I_o(h) \left[\frac{1}{\Delta R_o} - \frac{1}{\Delta R_j}\right] \ ; \ \alpha_{eo} < 10^\circ$$

Since for large R_o, the effective angle, α_{ej} would bear the same relationship to the angle parameters of ray path R_j as α_{eo} does to ray path R_o, the effective angles may be related by the crude approximation:

$$\alpha_{eo} - \alpha_{ej} \cong \phi_{je}$$

The range difference error then is approximated by

$$\Delta D_{jo} \cong \frac{I_o(\infty)\ \phi_{je} \cot\beta \csc\beta}{1 - \phi_{je} \cot\beta} \tag{9}$$

$$\text{where } \beta = \begin{cases} \alpha_o, & \alpha_o > 10^\circ \\ \alpha_{eo}, & \alpha_o < 10^\circ \end{cases} \quad \text{and } h_t > 30 \text{ km}$$

In terms of $I_o(\infty)$ and ΔR_o, the range difference error may be expressed by

$$\Delta D_{jo} \cong \frac{I_o(\infty)}{\sin\beta} \frac{\phi_{je}\cot\beta}{1-\phi_{je}\cot\beta} = \frac{\Delta R_o\,\phi_{je}}{\tan\beta-\phi_{je}} \tag{10}$$

$$\therefore \Delta D_{jo_{max}} \cong \frac{\Delta R_o\,\phi_j}{\sin\beta-\phi_j} \quad \text{for } \beta<30^\circ \text{ or } 530 \text{ mr}$$

$$\cong \frac{\Delta R_o\,\phi_j}{\frac{I_o(\infty)}{\Delta R_o}-\phi_j} \cong (\Delta R_o)^2 \quad \frac{B_{jo}}{r_o I_o(\infty)} \tag{11}$$

which holds for $I_o(\infty) >> \Delta R_o\,\phi_j$ or in most cases.

$$\therefore \Delta D_{jo_{max}} \cong 1.92\,(\Delta R_o)^2 \times 10^{-3} \text{ meters for } \Delta R_o \text{ in meters} \tag{12}$$

$\beta<30^\circ$, a baseline of 30.48 km (100 k ft) and $I_o(\infty) \doteq 2.5 \times 10^{-3}$km (5 year mean for Cape Canaveral).

The last expression is a good approximation for angles below 30°. For angles between 30° and 85° it will hold within an order of magnitude (R_o large) since, at 85°, $\tan\beta$ is an order of magnitude larger than $\sin\beta$. This expression therefore may be used to determine the upper bound on the range difference errors expected for the entire volume of space of interest to Mistram. The maximum values being obtained for targets along the extended baseline, B_{jo}

For an elevation angle of 200 mr (11.5°) the average of $I_o(\infty)$ for Cape Canaveral is, using (2),

$$\Delta R_o \cong \frac{2.5}{\sin(0.2 \text{ rad})} = 12.6 \text{ meters.}$$

The values of range error tabulated above (tables 2, 3) shows for this elevation angle,

$8.6m < \Delta R_o < 13.1$ meters.

For this case, using (9) and a baseline B_{jo} of 30.48 km. (100 k. ft.)

$\Delta D_{jo} = 0.30$ meters

or using (12)

$\Delta D_{jo} = 0.30$ meters.

The errors calculated are for an elevation angle of 11.5° and applies for ranges greater than 150 km. At this elevation angle, the range error varies from about 8 meters to about 13 meters. The corresponding range difference error varies from 0.12 meters to 0.33 meters. In both cases both the average expected errors and the day to day changes in the average error far exceed the maximum allowed errors of $\Delta R_{o_{max}} = 0.12$ meters and $\Delta D_{jo_{max}} = 0.009$ meters (table 1).

The effect of local inhomogeneities in the atmospheric index of refraction on ΔR_o was noted above. The same holds for ΔD_{jo}. An inhomogeneity such as a cloud that lies along the ray path between either the remote site and the missile or between the central station and the missile and not along the other ray path will cause a change in ΔD_{jo}.

A change in ΔD_{jo} of about 0.01 meters would result if an inhomogeneity occurs of the same size and average index change as computed above. This change in ΔD_{jo} is also of the same order of magnitude as the maximum allowed range difference errors. Larger disturbances such as frontal movements will cause even a more drastic change in ΔD_{jo}.

4.0 Range Rate and Range Difference Rate Errors

Errors in range rate and range difference rate will also be caused by atmospheric refraction. The errors are caused by the rate of change of the range error caused by the atmosphere.

$$\frac{dR_o}{dt} = \frac{d}{dt}\left(R_{SR} + \Delta R_o\right) = \frac{dR_{SR}}{dt} + \frac{d}{dt}\left(\Delta R_o\right)$$

The range rate error then is

$$\Delta\left(\frac{dR_o}{dt}\right) = \frac{d}{dt}\left(\Delta R_o\right)$$

and is related to both the rate of change of the target along a particular ray path (β = constant) and the rate of change of β. For motion along the ray path and a target height greater than 30 km, the range rate error is negligible. The error in this case is negligible because the change in range error per unit change in range is negligible for $h_t > 30$ km. The range rate error then may be considered due to changes in β.

$$\frac{d(\Delta R_o)}{dt} \cong \frac{d}{dt}\left(\frac{I_o(h)}{\sin\beta}\right) = -\Delta R_o \cot\beta \frac{d\beta}{dt} \quad \text{meters/sec} \tag{13}$$

for ΔR_o in meters and $h_t > 30$ km.

Similarly, for range difference rate errors due to refraction,

$$\frac{d}{dt}\left(\Delta D_{jo}\right) \cong \frac{d}{dt}\left(1.92\,(\Delta R_o)^2 \times 10^{-3}\right) = -2\Delta D_{jo} \cot\beta \frac{d\beta}{dt} \quad \text{meters/sec} \tag{14}$$

for ΔD_{jo} in meters and $h_t > 30$ km.

The rate errors described above are for a spherically symmetric atmosphere. These estimates do not include the rate errors caused by the motion of the ray path through inhomogeneities in the atmosphere.

5.0 The Effect of Range and Range Difference Errors

The range errors also contribute to errors in target position. A detailed analysis of position errors is impossible without a lengthy consideration of the use of the Mistram outputs. The general effect of errors in R_o and D_{jo} may be inferred by considering the magnitude of the position error, L, depicted in figure 3.

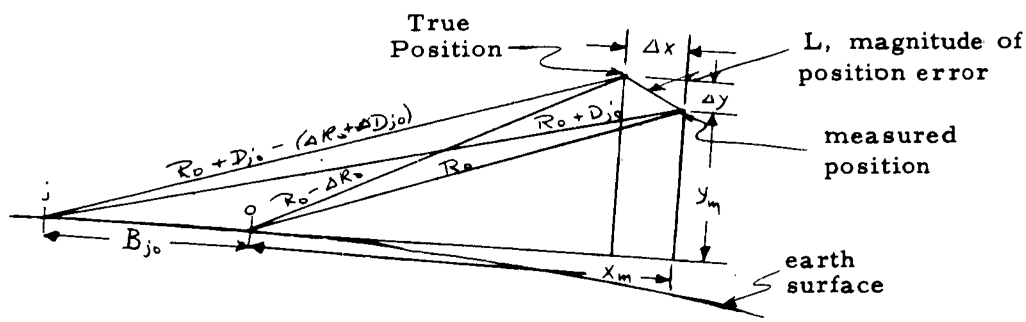

Figure 3

The measured distances from the central station to the target, x_m and y_m are

$$x_m = \frac{2R_o D_{jo} + (D_{jo})^2 - (B_{jo})^2}{2 B_{jo}} \tag{15}$$

$$y_m = \sqrt{R_o^2 - x_m^2} \tag{16}$$

where R_o is large, the rays are assumed to propagate in straight lines, and the target is assumed to be above the extended base line. The position errors are related to ΔR_o and ΔD_{jo} by

$$|\Delta x| = \left| \frac{\Delta R_o \ D_{jo} + \Delta D_{jo}(R_o + D_{jo})}{B_{jo}} \right| \tag{17}$$

$$|\Delta y| = \left| \frac{R_o \Delta R_o - x_m \Delta x}{y_m} \right| \tag{18}$$

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2} \quad . \qquad (19$$

The magnitudes of the position error, L, for the case computed above is

$L = 15.13$ meters

where $|\Delta x| = 14.12\text{m}$, $|\Delta y| = 5.44\text{m}$

$x_m = 146.41$ km, $y_m = 32.60$ km

$\Delta R_o = 12.6\text{m}$, $\Delta D_{jo} = 0.3_m$

$R_o = 150$ km, $D_{jo} = 30.48$ km

$30.48 \cos(200\text{ mr}) = 29.87$ km, $B_{jo} = 30.48$ km.

The asymptotic behavior of the position errors for large R_o may be deduced from the above expression. For large R_o, ΔR_o, and D_{jo} are essentially constant.

$$\therefore x_m = \frac{2 R_o D_{jo} + (D_{jo}^2 - B_{jo}^2)}{2 B_{jo}} \sim R_o \left(\frac{D_{jo}}{B_{jo}}\right) = k_1 R_o \qquad (20)$$

$$y_m = \sqrt{R_o^2 - x_m^2} \sim R_o \sqrt{1 - \left(\frac{D_{jo}}{B_{jo}}\right)^2} = k_2 R_o \qquad (21)$$

$$|\Delta x| = \frac{\Delta R_o D_{jo} + \Delta D_{jo} (R_o + D_{jo})}{B_{jo}} \sim R_o \left(\frac{\Delta D_{jo}}{B_{jo}}\right) = k_3 R_o \qquad (22)$$

$$|\Delta y| = \frac{R_o \Delta R_o - x_m \Delta x}{y_m} \sim \frac{R_o (\Delta R_o - k_1 k_3 R_o)}{k_2 R_o} \sim - \frac{k_1 k_3}{k_2} R_o \qquad (23)$$

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2} \sim R_o k_3 \sqrt{1 + \left(\frac{k_1}{k_2}\right)^2} = k_4 R_o \qquad (24)$$

The magnitude of the position error therefore asymptotically approaches a linear increase with R_o. For the case computed above, L increases linearly with R_o for $R_o > 15{,}000$ km

The errors in range difference and position have been considered in detail for the target along the extended base line, $\phi_{je} = \phi_j$. In general, the target will be at some angle with respect to the base line. The effective great circle arc between the remote site and the control station will be smaller or $\phi_{je} < \phi_j$.

From (10), the range difference error approximately is

$$\Delta D_{jo} = \frac{(\Delta R_o)^2 \ \phi_{je}}{I_o(\infty)} \propto \phi_{je}$$

Similarly, from (22)

$$|\Delta x| \sim \left(\frac{\Delta D_{jo}}{B_{jo}}\right) R_o \propto \phi_{je}$$

From (23)

$$|\Delta y| \sim - \frac{k_1 k_2}{k_2} R_o = - \frac{k_1}{k_2} \left(\frac{\Delta D_{jo}}{B_{jo}}\right) R_o \propto \phi_{je}$$

Therefore,

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2} \propto \phi_{je}$$

In the extreme case, when the range difference error is negligible, $\phi_{je} = 0$, the magnitude of the position error is due to ΔR_o alone and independent of range as

$$|\Delta x| \sim \Delta R_o \left(\frac{D_{jo}}{B_{jo}}\right)$$

$$|\Delta y| \sim \Delta R_o \sqrt{1 - \left(\frac{D_{jo}}{B_{jo}}\right)^2}$$

$$L \sim \Delta R_o$$

The Mistram system, however, requires data from a remote site, central station pair along the other baseline, to estimate position in three dimensions. The error from that station will, in general, be a maximum, $\phi_{je} \cong \phi_j$, and the composite, three dimensional error will asymptotically approach a linear increase with R_o.

6.0 Conclusions

The relative range errors and range difference errors decrease with increasing distance from the central station. The magnitude of the position error, however, increases with distance from the central station. The average refractive errors exceed the maximum allowed errors (Table 1) by one to three orders of magnitude. Refractive index corrections must be applied to the Mistram outputs to maintain refractive errors less than the maximum allowable overall system errors.

The daily variations in the range and range difference errors due to changing atmospheric conditions also may exceed the maximum allowable errors by one to two orders of magnitude. Errors due to local inhomogeneities may be of the same orders of magnitude as the specified

maximum allowable errors. To maintain the specified accuracy, the Mistram system must periodically sample the effects of the atmosphere and correct the output data. The refraction corrections must also be applied to each ray path separately.

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References

1. Introduction to Mistram , A Precision Missile Trajectory Measurement System. General Electric - DSD.

2. Thayer, G. D. and Bean, B. R., An Analysis of Atmospheric Refraction Error of Phase Measuring Radio Tracking Systems. Part I., NBS Report No. 7254, June 5, 1962.

3. Crane, R. K., Methods for Calculating Errors Due to Atmospheric Refraction. MITRE TM-3423, November 1962.

4. Bean, B. R. and Thayer, G. D., CRPL Exponential Reference Atmosphere, NBS Monograph 4, October 29, 1959.

5. Chapter 13, Handbood of Geophysics for Air Force Designers, 1st Edition, Geophysics Research Directorate, AFCRC.

6. Bean, B. R. and Cahoon, B. A., Effect of Atmospheric Horizontal Inhomogeneity Upon Ray Tracing, J. Research of NBS, 63D, No. 3, 287-292 (Nov. - Dec. 1959).

7. Bean, B. R., Climatology of Ground-Based Radio Ducts, J. Research of NBS, 63D, No. 1, 29-34(July-Aug. 1959).

Hq. ESD, L.G. Hanscom Field, Bedford, Mass.
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V. Secondary Rpt. No. TM-3422
VI. In ASTIA collection

exceed the maximum allowable errors for Mistram. Refractive index corrections must be applied to the Mistram System to reduce the errors to the same magnitude as the maximum allowable errors.

IV. Crane, R.K.
V. Secondary Rpt. No. TM-3422
VI. In ASTIA collection